Beginning with a distillation of the Maxwell postulates, the authors move on first to the dielectric and magnetic response characteristics of actual materials and the band diagrams of diverse crystals, and then to a comprehensive overview of the electromagnetic characteristics of periodic distributions of matter in space. Homogenizable as well as nonhomogenizable periodic composites are described, along with a useful introduction to transformation optics.

Prof. Akhlesh Lakhtakia
Pennsylvania State University, USA

This entry-level book is suitable for everyone interested in understanding and mastering metamaterial design. Starting from the necessary details of basic principles of electromagnetism, it develops all the way to illustrate implementations of sophisticated numerical models and metamaterial homogenization approaches. Care is taken to underline physical insights behind mathematical notions.

Prof. Anatoly Zayats
King’s College London, UK

I enjoyed reading the book edited by Felbacq and Bouchitté. It is a solid reference for the rigorous modeling and design of metamaterials, with special emphasis on the mathematical aspects. It will serve as a great reference for the field and as an inspiration for scientists and students entering this area of research.

Prof. Andrea Alù
University of Texas, USA

This book covers the fundamental physics, mathematics, and numerics necessary for entering the field of metamaterials. It presents advanced mathematical methods in a self-consistent way, along with numerous examples. It focuses on electromagnetic waves but is also useful in studying other types of metamaterials. It presents the structure of Maxwell equations, discusses the homogenization theory in detail, and includes important problems on resonance. It has an entire section devoted to numerical methods (finite elements, scattering theory), which motivates a reader to implement them. It offers numerous interesting examples at the forefront of research. The book is not written as a collection of independent chapters but as a textbook with a strong pedagogical flavor.

Didier Felbacq is full professor at the University of Montpellier, France. He graduated in mathematics and physics from the Ecole Centrale de Marseille, France, and Aix-Marseille University, France, respectively. His current research activities cover electron transport in transistors for terahertz emission and detection, second harmonic emission in photonic crystals, excitons in 2D materials, quantum and thermal metamaterials, and modeling in biology.

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Metamaterials
Modeling and Design
Metamaterials
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edited by
Didier Felbacq
Guy Bouchitté
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Preface

The domain of metamaterials now covers many areas of physics: electromagnetics, acoustics, mechanics, thermics, and even seismology. Huge literature is now available on the subject but the results are scattered. Although many ideas and possible applications have been proposed, which of these will emerge as a viable technology will only unfold with time. This book is concerned with electromagnetic waves only and deals essentially with the hard science, mathematical and numerical, behind the often spectacular, but somewhat oversold, possible applications of metamaterials. In a rapidly evolving field, with lots of would-be revolutions, spending too much pages on the zoology of metamaterials would certainly condemn this book to a rapid obsolescence. By contrast, the theoretical and numerical methods presented here are the basis upon which future trends will be built.

The first chapter is a survey of Maxwell’s equations and their main properties. After a short historical introduction, potentials and conservation laws are addressed. Then comes a brief presentation of the formulation of Maxwell equations using differential forms. Finally, causality and its consequences are addressed.

Chapter 2 provides the elements of the physics of materials required to bridge semiconductor and metal sciences with electromagnetism, and Chapter 3 is a general reflection upon the notion of averaging and the definition of effective properties.

Chapter 4 is a crash course on basic principles of transformation optics. Simple examples in cylindrical geometry are given using radial transformations that show the unifying power of the concept: mapping an open domain on a bounded domain, perfectly matched layers, invisibility cloaks, and superlenses. Some numerical
simulations are presented as an illustration including cloaks of arbitrary shapes and mimetism.

Chapter 5 is concerned with wave propagation in periodic media. The theory of Bloch waves is described in detail. The situation where the medium does not cover the entire space is addressed, because in that situation the boundary of the periodic medium is decorated with evanescent modes. Evanescent waves are then investigated. They are shown to be a complexification (in the mathematical meaning) of the Bloch spectrum.

Chapter 6 tackles the problem of diffraction of electromagnetic waves by a bidimensional grating. A new formulation based on finite element method is proposed. A lot of academic cases and more challenging cases are given for highlighting both the versatility and the powerfulness of the method described in this chapter. The second part of the chapter is devoted to the method of multiple scattering, which is presented for a collection of parallel cylinders.

Chapter 7 is the first chapter devoted to effective properties of metamaterials. Periodic structures are considered and the period of the materials are small with regard to the wavelength of the incoming wave. Besides, the materials are supposed to be of low contrast: this is the framework of soft problems. Closed formulae are given in some academic cases such as small spherical and small circular cylindrical inclusions. A special attention is drawn on spherical inclusions and the mixing formulae (Rayleigh, Maxwell Garnett, Bruggeman) are compared to the two-scale theory.

Chapter 8 addresses the homogenization of highly contrasted objects. The first situation investigated is that of a periodic collection of thin metallic wires. It is shown that the effective medium obtained is dispersive and has a plasmonic resonance. In the second part, the theory is extended to deal with finite-length rods. It is proven that the effective medium becomes spatially dispersive. The chapter closes with numerical investigations of the properties of the effective medium.

The final chapter is also devoted to homogenization theory. It deals with the possibility of homogenizing metamaterials for
frequency above the first band and taking into account the Mie resonances. Bidimensional resonant dielectric metamaterials are addressed and the onset of an effective magnetic activity is proven.

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