Thanks to the pioneering works of Ashkin and co-workers, optical tweezers (OT) have become an invaluable tool for myriad studies throughout the natural sciences. Their success relies on the fact that they can be considered as exceptionally sensitive transducers able to resolve pN forces and nanoscale displacements, with high temporal resolution, down to microseconds. No wonder that more than 65,000 related articles discuss their application in the study of a wide range of biological phenomena, such as measuring the compliance of bacterial tails, the forces exerted by a single motor protein, the mechanical properties of human red blood cells, and those of individual biological molecules.

Microrheology is a branch of rheology, but it works at micrometre length scales and with microlitre sample volumes. Therefore, microrheology techniques are revealed to be very useful tools for all those rheological/mechanical studies where rare or precious materials are employed, such as in biological and biomedical studies.

The aim of this book is to provide a pedagogical introduction to the physics principles governing both the optical tweezers and their application in the field of microrheology of complex materials. This is achieved by following a linear path that starts from a narrative introduction of the "nature of light", followed by a rigorous description of the fundamental equations governing the propagation of light through matter. Moreover, some of the many possible instrumental configurations are presented, especially those that better adapt to perform microrheology measurements. In order to better appreciate the microrheological methods, with optical tweezers explored in this book, informative introductions to the basic concepts of linear rheology, statistical mechanics, and the most popular microrheology techniques are also given. Furthermore, an enlightening prologue to the general applications of optical tweezers different from rheological purposes is provided at the end of the book.

Manlio Tassieri is a lecturer within the Division of Biomedical Engineering at the University of Glasgow. He is a council member of the British Society of Rheology. Graduating as a chemical engineer from the Department of Chemical Engineering, the University of Naples “Federico II”, in 2000, he developed two novel rheo-optical methods for determining interfacial tension in disperse polymer blends. In 2003 he decided to follow his aspiration to become an academic researcher. To do this, he embarked on research in the field of microrheology of semiflexible biopolymers at the School of Physics and Astronomy of the University of Leeds, from where he graduated with a PhD in 2007. Following his PhD, he held a postdoctoral research position in the Polymer Science and Technology IRC at the University of Leeds, collaborating in the Microscale Polymer Processing project. In 2010 he was awarded a Royal Academy of Engineering Research Fellowship to combine microrheological techniques with microfluidic devices. Dr Tassieri has contributed to the field of microrheology with a number of research articles published in reputed journals.
Microrheology with Optical Tweezers
Microrheology with Optical Tweezers
Principles and Applications

edited by
Manlio Tassieri
To Loredana
Contents

Foreword xiii
Editor’s Preface xv

PART I INTRODUCTION

1 General Introduction to Optical Tweezers and Their Applications 3
Manlio Tassieri
1.1 Introduction 3

PART II OPTICAL TWEEZERS

2 The Nature of Light 9
R. Mike L. Evans
2.1 A Condensed History of Optics 9
2.2 Wave Physics 14
2.3 Electromagnetism 19
2.3.1 Fields 20
2.3.2 Coulomb’s Law and Gauss’s Law 20
2.3.3 Faraday’s Law 24
2.3.4 Ampère’s Law and Displacement Current 26
2.3.5 Electromagnetism and Light 29
2.4 Interaction of Light with Matter 32
2.5 Interaction of Light with Metals 35
2.6 Photons and Lasers 37

3 Geometrical Optics 41
Alison Yao
3.1 Introduction 41
3.2 Maxwell’s Equations 42
3.3 From Maxwell’s Equations to the Wave Equation 44
  3.3.1 Wave Equations in a Vacuum 44
3.4 Solutions to the Wave Equation 48
  3.4.1 Plane Wave Solutions 48
  3.4.2 Properties of Plane Wave Solutions 49
  3.4.3 Polarization 50
  3.4.4 Wave Equations in a Dielectric (Non-Conducting) Medium 52
3.5 Reflection and Transmission at an Interface 53
  3.5.1 Normal Incidence 55
  3.5.2 Oblique Incidence 59
    3.5.2.1 Magnitudes of the transmitted and reflected fields 61
    3.5.2.2 \( E_{\text{Perpendicular}} \) to plane of incidence 61
    3.5.2.3 \( E_{\text{Parallel}} \) to plane of incidence 64
    3.5.2.4 Brewster’s angle and total internal reflection 66
3.6 Beam Solutions to the Wave Equation 67
  3.6.1 Gaussian Beam Solutions 68
  3.6.2 Higher-Order Solutions 75

4 Optical Forces 81
  Michael P. Lee and David B. Phillips
  4.1 Introduction 81
  4.2 Gradient Forces 84
  4.3 Ray Optics Description of Optical Tweezers 85
  4.4 The Electric Dipole Description of Optical Tweezers 94
  4.5 Generalized Lorenz-Mie Theory and Numerical Simulation 97
  4.6 Optical Torques 98
  4.7 Conclusions 100

5 Optical Tweezers Configurations 103
  Graham M. Gibson
  5.1 Introduction 103
  5.2 Different Lasers Wave-Lengths for Different Applications 105
Contents

5.3 Objectives 107
5.4 Sample Holders 108
5.5 Controlling the Trap Position 109
  5.5.1 Steering Mirrors (Motorised) 110
  5.5.2 Acousto-Optic Deflectors 112
  5.5.3 Spatial Light Modulators 112
  5.5.4 Hologram Calculation 115
    5.5.4.1 Gerchberg Saxton 115
    5.5.4.2 Gratings and lenses 116
5.6 Measuring Position and Force 120
  5.6.1 Quadrant Photodiodes 121
  5.6.2 Digital Video Cameras 123
  5.6.3 Calibration Using Stokes’ Drag Method 125
  5.6.4 Calibration Using Equipartition Theorem 127
  5.6.5 Calibration Using Power Spectrum Analysis 127
  5.6.6 Measuring the Accuracy of Particle Position and Force in Optical Tweezers 128
  5.6.7 Stereoscopic Particle Tracking 131
5.7 Conclusions 133

PART III MICRORHEOLOGY

6 Introduction to Linear Rheology 137
   Manlio Tassieri
   6.1 Introduction 137
   6.2 Linear Rheology for Simple Shear 139
   6.3 Simple Mechanical Models of Linear Viscoelastic Behaviour 145

7 Statistical Mechanics and Diffusion Processes 155
   Adrian Baule
   7.1 Introduction 155
   7.2 Diffusion Processes 156
     7.2.1 Velocity of a Brownian Particle 158
     7.2.2 Particle Position 162
     7.2.3 Correlations and Response 164
       7.2.3.1 Response functions 167
     7.2.4 Simulation of Langevin Equations 169
7.2.4.1 Error estimation 171
7.2.4.2 Determining the probability density function 173
7.3 Probability Density Functions 174
  7.3.1 Itô’s Formula and the Fokker-Planck Equation 174
  7.3.2 Ornstein-Uhlenbeck Process 177
  7.3.3 The Multivariate Case 179
7.4 The Overdamped Limit 181
  7.4.1 Escape from a Metastable Potential 184
7.5 Microrheology and the Generalized Langevin Equation 187
  7.5.1 Measuring Viscosity in Newtonian Fluids 188
  7.5.2 Viscoelasticity 189

8 Most Popular Microrheology Techniques 193
  Aristeidis Papagiannopoulos

8.1 Introduction 193
8.2 Theoretical Background of Microrheology 194
8.3 Video Particle Tracking Microrheology 200
8.4 Microrheology with Single Light Scattering 209
8.5 Microrheology with Diffusing Wave Spectroscopy 211
8.6 Microrheology with Magnetic Tweezers 214

9 Microrheology with Optical Tweezers 219
  Manlio Tassieri

9.1 Introduction 219
9.2 Optical Tweezers Calibration 220
  9.2.1 Spatial Calibration 221
  9.2.2 Elastic Constant Calibration 222
9.3 Microrheology with Static Optical Tweezers 223
  9.3.1 Solving a Generalised Langevin Equation for Static OT 223
  9.3.2 Data Analysis 228
    9.3.2.1 Interpolation artefacts 228
    9.3.2.2 Noise 232
9.4 Active Microrheology with Optical Tweezers 235
  9.4.1 Entraining Flow Field 236
  9.4.2 Flipping Bead 243
9.5 A Rheological Interpretation of Optical Tweezers 249
## Contents

**PART IV  REVIEW ON OPTICAL TWEEZERS APPLICATIONS**

10  **Optical Tweezers Outwith Microrheology**  
*Richard W. Bowman*  
10.1 Introduction  
10.2 Optical Momentum  
10.3 Statistical Mechanics  
10.4 Optical Binding  
10.5 Counterpropagating Traps  
10.6 Single Molecule Studies  
10.7 Scanning Probe Microscopy  
10.8 Vacuum Trapping and Cooling  
10.9 Conclusions

**Appendix: Evaluating the Fourier Transform**  
*R. Mike L. Evans*  
A.1 Introduction  
A.2 Transforming from Time to Frequency with Minimal Artefacts

*References*  
*Index*
Foreword

Following more than a decade of related pioneering research, Arthur Ashkin and co-workers (Ashkin et al., 1986) in 1986 published the seminal paper that inspired the field of optical tweezers. Optical tweezers use tightly focussed beams of laser light to trap transparent, micrometer-sized particles, typically suspended in a fluid. That these micrometer-sized particles could be trapped and manipulated using only a single laser beam meant that optical tweezers could easily be implemented as a simple addition to a conventional microscope. This simplicity of construction makes optical tweezers an accessible tool for further research. The ability to use optical tweezers for measurements of picoNewton forces and nanometer displacements is of particular interest to the biophysics community. For example, by using optical tweezers, Block and co-workers were able study the properties of individual motor proteins (Svoboda et al., 1993).

The physical sciences too have embraced optical tweezers and in 1995 Rubinsztein-Dunlop and co-workers (He et al., 1995) demonstrated that by using specially modified laser beams they could transfer angular momentum between the light and the particle, causing the particle to spin. This conversion of an optical tweezer to an optical spanner both answered fundamental questions about the nature of light and suggested new approaches to the creation of optically driven micromachines.

A major technical advance in optical tweezers was championed by David Grier and co-workers (Grier, 2003). They used a single laser but added a spatial light modulator to give the user real-time control of both the structure of the trapping beam and indeed the number of independent traps. This simple addition of one extra optical component creates holographic optical tweezers capable of
simultaneously trapping many particles at defined positions within a sample volume.

The use of optical tweezers to measure both forces and position relies on their ability to measure the position of the centre of mass of the trapped probe-particle to a small fraction of the optical wavelength. Initially these measurements used quadrant photodiodes, aligned to the axis of the particle, to measure the slight deflection in the transmission of the trapping laser beam. More recently similar performance has been achieved using high-speed video cameras, which are both easier to align and capable of tracking multiple particles simultaneously.

Being able to measure the higher frequency vibrations of the trapped particles means that the particle motion can be related to the viscoelastic properties of the fluid in which the particle(s) is (are) suspended. These measures of local viscosity, microrheology, have been applied within microfluidic channels and inter- or even intracellular environments and the tracking of multiple particles reveals the eigenmodes of particle networks and the precise nature of their hydrodynamic coupling and potential synchronisation.

As cameras, spatial light modulators, and computing technologies advance, the complexity of the systems that can be both controlled and probed will also advance. These advances will cover systems which encompass the synchronisation of motor proteins, the folding of complex molecules, and the creation of digital colloids.

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Editor’s Preface

The aim of this book is to provide a pedagogical introduction to the physics principles governing optical tweezers and their application in the field of microrheology.

This is achieved by following a linear path that starts from a narrative introduction of the “nature of light”, followed by a rigorous description of the fundamental equations governing the propagation of light through matter. In particular, the ability of light to exert a force on micrometer-sized objects is described, this property being the fulcrum of the optical tweezers’ working principles. Moreover, some of the many possible instrumental configurations are presented, especially those that better adapt to perform microrheology measurements.

In order to better appreciate the microrheological methods with optical tweezers explored in this book, informative introductions to the basic concepts of linear rheology, statistical mechanics, and the most popular microrheology techniques are also given.

Furthermore, an enlightening prologue to the general applications of optical tweezers different from rheological purposes is provided at the end of the book.

I am grateful to all the co-authors for their generous contributions to this book. I have a debt of gratitude to both Professor Jonathan M. Cooper and Professor Miles J. Padgett for their continuous guidance and support during the editing of this book. I would like to thank Niall P. MacDonald for the book cover. The book has been edited/written during the tenure of a Royal Academy of Engineering/The Engineering and Physical Sciences Research Council Research Fellowship. Finally, I wish to thank the University
Editor’s Preface

of Glasgow for the supportive and enthusiastic work environment that has facilitated the writing of this book.

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