The properties of strongly correlated electrons confined in two dimensions are a forefront area of modern condensed matter physics. In the past two or three decades, strongly correlated electron systems have garnered a great deal of scientific interest due to their unique and often unpredictable behavior. Two of many examples are the metallic state and the metal–insulator transition discovered in 2D semiconductors: phenomena that cannot occur in noninteracting systems. Tremendous efforts have been made, in both theory and experiment, to create an adequate understanding of the situation; however, a consensus has still not been reached.

Strongly Correlated Electrons in Two Dimensions compiles and details cutting-edge research in experimental and theoretical physics of strongly correlated electron systems by leading scientists in the field. The book covers recent theoretical work exploring the quantum criticality of Mott and Wigner–Mott transitions, experiments on the metal–insulator transition and related phenomena in clean and dilute systems, the effect of spin and isospin degrees of freedom on low-temperature transport in two dimensions, electron transport near the 2D Mott transition, experimentally observed temperature and magnetic field dependencies of resistivity in silicon-based systems with different levels of disorder, and microscopic theory of the interacting electrons in two dimensions. Edited by Sergey Kravchenko, a prominent experimentalist, this book will appeal to advanced graduate-level students and researchers specializing in condensed matter physics, nanophysics, and low-temperature physics, especially those involved in the science of strong correlations, 2D semiconductors, and conductor–insulator transitions.

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Strongly Correlated Electrons in Two Dimensions
Strongly Correlated Electrons in Two Dimensions

edited by
Sergey Kravchenko
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Properties of strongly correlated electrons confined in two dimensions are a forefront area of modern condensed matter physics. Two-dimensional (2D) electron systems can be realized on semiconductor surfaces (metal-insulator-semiconductor structures, heterostructures, quantum wells); other examples include electrons on a surface of liquid helium or a single layer of carbon atoms (graphene). In some of these systems, Coulomb repulsion between electrons is small compared to the kinetic energy of electrons; such systems can be well described by Fermi liquid theory introduced by Landau in 1956. However, when the energy associated with the Coulomb interactions becomes larger (sometimes by orders of magnitude or even more) than the Fermi energy, perturbation theories fail and one may expect novel states of matter to form.

In a zero magnetic field, idealized (noninteracting) 2D electrons were predicted by the “Gang of Four” (Abrahams, Anderson, Licciardello, and Ramakrishnan, 1979) to become localized in the limit of zero temperature, no matter how weak the disorder in the system. Weak interactions between electrons are expected to contribute to the localization (Altshuler, Aronov, and Lee, 1980). Therefore, it came as a surprise when the metallic (delocalized) state and the metal–insulator transition were observed in a 2D electron system formed in low-disordered silicon transistors (Kravchenko et al., 1994). Since then, a tremendous effort has been made, in both theory and experiment, to produce an adequate understanding of the situation; however, a consensus has still not been reached.

In the limit of very strong interactions, electrons are supposed to crystalize into a lattice to minimize their repulsion energy (Wigner, 1934). A classical Wigner crystal has indeed been realized for electrons on the surface of liquid helium. Although indications
exist that Wigner crystallization also occurs in very dilute electron systems on semiconductor surfaces (where the crystal should be quantum), the “smoking-gun evidence” has never been obtained.

These are just two examples of many outstanding unsolved problems in the physics of strong correlations in two dimensions.

This book, intended for advanced graduate students and researchers entering the field, contains six chapters. In Chapter 1, a review is given on the recent theoretical work exploring quantum criticality of Mott and Wigner–Mott transitions. The authors argue that the most puzzling features of the experiments find natural and physically transparent interpretations based on this perspective.

Chapter 2 is devoted to experiments on very clean and very dilute 2D electron systems. Experimental results on the metal–insulator transition and related phenomena in such systems are discussed. Special attention is given to recent results for the strongly enhanced spin susceptibility, effective mass, and thermopower in low-disordered silicon transistors.

In Chapter 3, the author shows how spin and isospin degrees of freedom affect low-temperature transport in strongly interacting disordered 2D electron systems and explains experimentally observed temperature and magnetic field dependencies of resistivity in silicon-based systems.

In Chapter 4, recent experimental studies on the Mott transitions of layered organic materials are reviewed with an emphasis on quantum-critical transport. The authors show that in the vicinity of the Mott transition, different kinds of phases emerge, such as antiferromagnetic Mott insulators, quantum spin liquids, Fermi liquids, and unconventional superconductors.

Chapter 5 is a review of experimental results obtained on 2D electron systems with different levels of disorder. In particular, the author shows that sufficiently strong disorder changes the nature of the metal–insulator transition. Comprehensive studies of the charge dynamics are also reviewed, describing evidence that the metal–insulator transition in a 2D electron system in silicon should be viewed as the melting of the Coulomb glass.

Finally, in Chapter 6, a microscopic theory of a strongly correlated 2D electron gas is presented. The authors suggest an explanation of the divergence of the effective electron mass experimentally
observed in silicon-based 2D structures. Possible condensation of fermions in 2D electron systems, closely related to the condensation of bosons in superconductors or in superfluids, is also discussed.

I hope that this book will stimulate further developments in the physics of strongly correlated electrons in two dimensions and lead to many discoveries of yet unforeseen new physics.

Sergey Kravchenko